The Integral Equation

Introduction

This chapter introduces the theoretical framework of this study, starting with the description of the classical EFIE [1] and MPIE formulations [2]-[4]. In order to clarify how the Green's function is obtained the transmission line model is also included. Since different formulations for the Mixed Potential Integral Equation are possible, their features with respect to our goal are reported.

Electric Field Integral Equation Formulation

Consider a medium consisting of $n+1$ dielectric layers separated by n planar interfaces parallel to the xy plane of a Cartesian coordinate system and located at $z = z_i$, $i = 1, 2, ..., n$ as illustrated in Fig. 1. Each layer is characterized by a permittivity ε_i and permeability μ_i. Before writing the electric field integral equation for the above mentioned problem we have to make a few assumptions. First of all we consider an object embedded in a multilayered media, as sketched in Fig. 1, and we assume that it occupies p layers (named $L = l_1, l_2..., l_p$) with $1 \leq p \leq n+1$. Let S_i be the surface of the object and $\hat{n}_i$ be the unit vector normal to S_i. By using the equivalence principle we derive

$$\mathbf{M}_s = -\hat{n}_m \times (\mathbf{E}^i_m + \mathbf{E}^s_m[\mathbf{J}_s; \mathbf{M}_s])_{S_+} \quad (1)$$

$$\mathbf{M}_s = \hat{n}_m \times (\mathbf{H}^i_m + \mathbf{H}^s_m[\mathbf{J}_s; \mathbf{M}_s])_{S_+} \quad (2)$$

Moreover, if the medium is linear, we may express the fields due to arbitrary current distributions as

$$\mathbf{E} = \int_S \underline{\mathbf{G}}^{Ee} \cdot \mathbf{J} dS + \int_S \underline{\mathbf{G}}^{Em} \cdot \mathbf{M} dS \quad (3)$$

$$\mathbf{H} = \int_S \underline{\mathbf{G}}^{He} \cdot \mathbf{J} dS + \int_S \underline{\mathbf{G}}^{Hm} \cdot \mathbf{M} dS \quad (4)$$

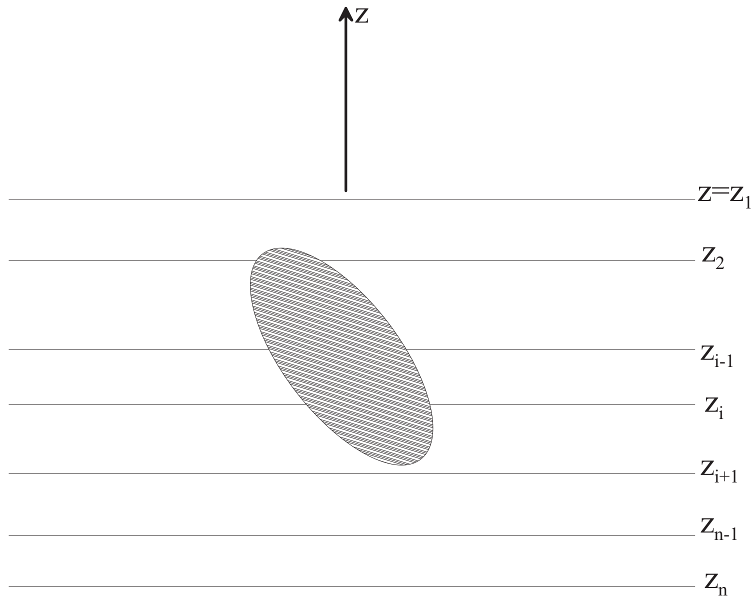

Figure 1: Generic multilayered medium geometry.

where $\underline{\mathbf{G}}^{PQ}$ is the dyadic Green's function relating the P-type fields at $\mathbf{r}$ to the Q-type currents at $\mathbf{r}\,'$, where $\mathbf{r}$ and $\mathbf{r}\,'$ are the radial distance of source and observation, respectively. For the sake of simplicity we consider only the case of a PEC object embedded in a multilayered medium. The boundary condition on S_i in this case is reduced to

$$-\hat{n}_m \times \mathbf{E}_m^s(\mathbf{r}) = \hat{n}_m \times \mathbf{E}_m^{inc}(\mathbf{r}) \qquad \mathbf{r} \quad on \quad S_m, m \in L \tag{5}$$

where $\mathbf{E}_m^{inc}$ is the electric field in the $m-th$ layer in the absence of the object, and $\mathbf{E}_m^s$ is the scattered electric field in the $m-th$ layer. The $\mathbf{E}_m^s$ and the magnetic scattered field H_m^s for the described problem can be expressed as

$$-\mathbf{E}_m^s(\mathbf{r}) = \sum_{i \in L} \left[j\omega \mathbf{A}^{mi}(\mathbf{r}) + \nabla \Phi^{mi}(\mathbf{r}) \right] \tag{6}$$

$$\mathbf{H}_m^s(\mathbf{r}) = \frac{1}{\mu_m} \nabla \times \sum_{i \in L} \mathbf{A}^{mi}(\mathbf{r}) \tag{7}$$

where the magnetic vector potential in the $m-th$ layer due to a current $\mathbf{J}$ in the $i-th$ layer and the corresponding scalar potential Φ^{mi} are given as

$$\mathbf{A}^{mi}(\mathbf{r}) = \int_{S_i} \underline{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}\,') \cdot \mathbf{J}(\mathbf{r}\,') dS\,' \tag{8}$$

$$\Phi^{mi}(\mathbf{r}) = \frac{j\omega}{k_m^2} \nabla \cdot \mathbf{A}^{mi}(\mathbf{r}) \tag{9}$$

The dyadic Green's function, which represents the magnetic vector potential in region m due to a unit-strength, arbitrarily oriented dipole in region i, can be found simply by solving the inhomogeneous Helmholtz equation [5]

$$(\nabla^2 + k_m^2)\underline{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}\,') = -\mu_m \underline{\underline{I}}\delta(\mathbf{r} - \mathbf{r}\,') \tag{10}$$

Considering a horizontal, x-directed dipole, the Green's function takes the following forms, according to whether or not the z or the y component is chosen to accompany the primary component x [3].

$$\underline{\mathbf{G}}_A^i = (\hat{x}\hat{x} + \hat{y}\hat{y})\mathbf{G}_{xx}^i + \hat{z}\hat{x}\mathbf{G}_{zx}^i + \hat{z}\hat{y}\mathbf{G}_{zy}^i + \hat{z}\hat{z}\mathbf{G}_{zz}^i\,, \tag{11}$$

$$\underline{\mathbf{G}}_A^i = \hat{x}\hat{x}\mathbf{G}_{xx}^i + \hat{y}\hat{y}\mathbf{G}_{yy}^i + (\hat{x}\hat{y} + \hat{y}\hat{x})\mathbf{G}_{xy}^i + \hat{z}\hat{z}\mathbf{G}_{zz}^i\,, \tag{12}$$

Substituting (9) in (7) and then in (5) we obtain the so-called vector-potential EFIE.

$$\frac{j\omega}{k_m^2}\hat{n}_m \times \sum_{i\in L}\left(k_m^2 \int_{S_i} \underline{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}\,')\cdot \mathbf{J}(\mathbf{r}\,')dS\,' + \nabla\nabla\cdot \int_{S_i} \underline{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}\,')\cdot \mathbf{J}(\mathbf{r}\,')dS\,'\right) = \hat{n}_m \times \mathbf{E}_m^{inc}(\mathbf{r}) \tag{13}$$

By introducing one of the operators ∇ into the integral and then transferring it to act on the current, we obtain the desidered mixed-potential EFIE. It is not convenient to introduce both the operators ∇ into the integral because of the highly singular behavior of the dyadic kernel thus obtained [3].

MPIE Mixed Potential EFIE

Several studies have pointed out the advantages of the mixed potential formulation in solving antenna problems [6]-[7]. It would be possible to derive the desidered mixed potential form from 13 if the scalar potential could be expressed in terms of surface charge density $q(r)$. Based on 9 we can rewrite the scaler potential as

$$\Phi^{mi}(\mathbf{r}) = \frac{j\omega}{k_m^2}\int_{S_i}\left[\nabla\cdot \underline{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}\,')\right]\cdot \mathbf{J}(\mathbf{r}\,')dS\,' \tag{14}$$

Moreover, we have to transfer the divergence operator to act on the current, in view of the equation of continuity $\nabla\cdot\mathbf{J} = -j\omega q$. The desidered mixed potential form would be achieved if the scalar potential could be expressed in terms of surface charge density $q(\mathbf{r})$. Therefore, we have to transfer the divergence operator to act on the current, in view of the equation of continuity $\nabla\cdot\mathbf{J} = -j\omega q$. This can be done only if we can define a scalar function G_Φ^{mi}, such as

$$\frac{j\omega}{k_m^2}\nabla\cdot \underline{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}\,') = \frac{1}{j\omega}\nabla\,' G_\Phi^{mi}(\mathbf{r}|\mathbf{r}\,'). \tag{15}$$

For a stratified medium, in general, a G_Φ^{mi} that satisfies eq. 15 does not exist [8]. This can be attributed to the fact that the scalar potentials of point charges associated with vertical and horizontal dipoles in a layered medium are usually different [9]. Then, in order to achieve our goal, let us introduce a scalar function K_Φ^{mi} and a vector function $\mathbf{P}^{mi}$ according to

$$\frac{j\omega}{k_m^2}\nabla\cdot\underline{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}') = \frac{1}{j\omega}\nabla' K_\Phi^{mi}(\mathbf{r}|\mathbf{r}') + j\omega\mathbf{P}^{mi}(\mathbf{r}|\mathbf{r}'). \tag{16}$$

substituting (16) in (14) and introducing the dyadic kernel

$$\underline{\mathbf{K}}_A^{mi}(\mathbf{r}|\mathbf{r}') = \underline{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}') + \nabla\mathbf{P}^{mi}(\mathbf{r}|\mathbf{r}'). \tag{17}$$

The mixed potential representation for the fields E and H is then

$$\begin{aligned}\mathbf{E} = -j\omega\mu\int_{S_i}\underline{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')dS' + \frac{1}{j\omega\epsilon}\int_{S_i}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')q(\mathbf{r}')dS' +\\ \frac{1}{j\omega\epsilon}\nabla\int_{S_i}\mathbf{P}^{mi}(\mathbf{r}|\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')dS' + \int_{S_i}\underline{\mathbf{G}}_{Em}^{mi}(\mathbf{r}|\mathbf{r}')\cdot\mathbf{M}(\mathbf{r}')dS' +\\ \frac{\nabla}{j\omega}\oint_{C_i}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')\mathbf{J}(\mathbf{r}')\cdot\hat{u}_i dC' - \oint_{C_{i-1}}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')\mathbf{J}(\mathbf{r}')\cdot\hat{u}_{i-1}dC'\end{aligned} \tag{18}$$

$$\begin{aligned}\mathbf{H} = -j\omega\epsilon\int_{S_i}\underline{\mathbf{G}}_F^{mi}(\mathbf{r}|\mathbf{r}')\cdot\mathbf{M}(\mathbf{r}')dS' + \frac{1}{j\omega\mu}\int_{S_i}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')q_m(\mathbf{r}')dS' +\\ \frac{1}{j\omega\mu}\nabla\int_{S_i}\mathbf{P}^{mi}(\mathbf{r}|\mathbf{r}')\cdot\mathbf{M}(\mathbf{r}')dS' + \int_{S_i}\underline{\mathbf{G}}_{He}^{mi}(\mathbf{r}|\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')dS' +\\ \frac{\nabla}{j\omega}\oint_{C_i}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')\mathbf{M}(\mathbf{r}')\cdot\hat{u}_i dC' - \oint_{C_{i-1}}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')\mathbf{M}(\mathbf{r}')\cdot\hat{u}_{i-1}dC'\end{aligned} \tag{19}$$

where C_i and C_{i-1} are the contours formed by the intersection of the surface S_i with the interfaces $z = z_i$ and $z = z_{i-1}$, respectively. Consequently, using (18), the (5) reduces to

$$\begin{aligned}\hat{n_m}\times\sum_{i\in L}\Bigg(j\omega\int_{S_i}\underline{\mathbf{K}}_A^{mi}(\mathbf{r}|\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')dS' + \nabla\int_{S_i}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')q(\mathbf{r}')dS' +\\ \frac{\nabla}{j\omega}\oint_{C_i}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')\mathbf{J}(\mathbf{r}')\cdot\hat{u}_i dC' - \oint_{C_{i-1}}K_\phi^{mi}(\mathbf{r}|\mathbf{r}')\mathbf{J}(\mathbf{r}')\cdot\hat{u}_{i-1}dC'\Bigg) =\\ \hat{n_m}\times\mathbf{E}_m^{inc}(\mathbf{r})\end{aligned} \tag{20}$$

The unit vectors u_i and u_{i-1} are defined as $\hat{u}_i = \hat{t}_i \times \hat{n}_i$, where $\hat{n}_i$ is the normal vector to C_i and $\hat{t}_i$ is the tangent vector to C_i. Since K_Φ^{mi} and $\mathbf{P}^{mi}$ are not unique, the contour integrals, properly chosen, could be canceled out, thus obtaining the desidered mixed potential formulation. The choice of K_Φ^{mi} and $\mathbf{P}^{mi}$ gives rise to different formulations.

Transmission line model

Before discussing the possible MPIE formulations it is useful to derive the components of the Green's function either in (11) or (12). The transmission line model allows us to obtain a model in which the exponential form of the spectral domain Green's function can be easily pointed out. Since we are looking for a spectral domain form for the Green's function suitable for the GPOF formulation [10] the transmission line model seems to be the most useful one. Moreover, the analysis is easier when the problem is formulated in a transformed spectral domain, according to the following Fourier transform pair [1]

$$\mathcal{F}[f(r)] = \tilde{f}(k_\rho; z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(r)e^{jk_\rho\cdot\rho}dxdy \tag{21}$$

$$\mathcal{F}^{-1}[\tilde{f}(k_\rho; z)] = f(r) = \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{f}(k_\rho; z)e^{-jk_\rho\cdot\rho}dk_x dk_y \tag{22}$$

where $\rho = \hat{x}x + \hat{y}y$ and $k_\rho = \hat{x}k_x + \hat{y}k_y$. When we consider homogeneous layered media of infinite extend in any transverse (to z) plane, the spectral domain approach is the easiest way to determine the Green's function, since it reduces the original problem to solve an equivalent transmission line network, where each section corresponds to a different layer of the medium. [11], [12]. The medium is characterized by a z-dependent complex-valued permeability, in general, and permittivity dyadics $\underline{\underline{\mu}} = \underline{\underline{I_t}}\mu_t + \hat{z}\hat{z}\mu_z$, $\underline{\underline{\epsilon}} = \underline{\underline{I_t}}\epsilon_t + \hat{z}\hat{z}\epsilon_z$, where $\underline{\underline{I_t}}$ is the transverse unit dyadic. As a consequence of our assumptions we have made, we can separate the fields into their transverse and longitudinal parts, obtaining

$$H_z = \frac{1}{j\omega\mu}[\nabla_t \cdot (z \times \mathbf{E}_t) - M_z] \tag{23}$$

$$E_z = \frac{1}{j\omega\epsilon}[\nabla_t \cdot (\mathbf{H}_t \times z) - J_z] \tag{24}$$

$$-\frac{\partial \mathbf{E}_t}{\partial z} = j\omega\mu\left(\underline{\underline{\mathbf{I}_t}} + \frac{\nabla_t\nabla_t}{k^2}\right)\cdot(\mathbf{H}_t \times z) + \mathbf{M}_{te} \times z \tag{25}$$

$$-\frac{\partial \mathbf{H}_t}{\partial z} = j\omega\epsilon\left(\underline{\underline{\mathbf{I}_t}} + \frac{\nabla_t\nabla_t}{k^2}\right)\cdot(z \times \mathbf{E}_t) + z \times \mathbf{J}_{te} \tag{26}$$

where $\mathbf{M}_{te} = \mathbf{M}_t + z \times \frac{\nabla_t J_z}{j\omega\epsilon}$ and $\mathbf{J}_{te} = \mathbf{J}_t + z \times \frac{\nabla_t M_z}{j\omega\mu}$. Upon applying the Fourier transform pair defined in (22) to (26) and considering that the operator ∇ becomes

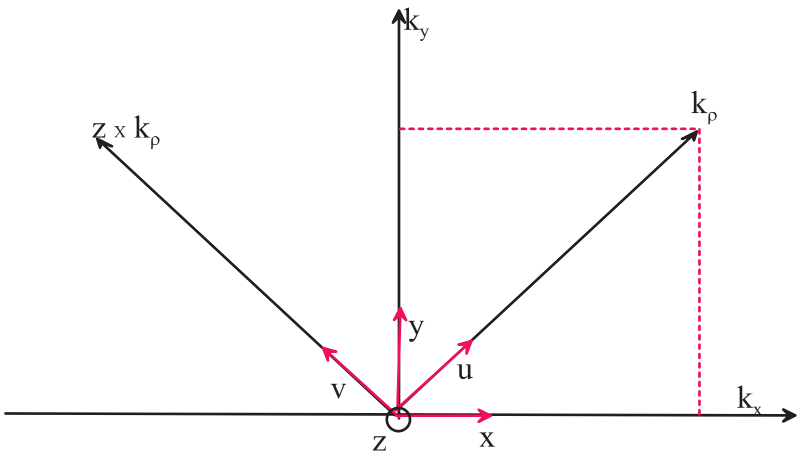

Figure 2: spectral-domain coordinate system.

$\nabla_t \leftarrow -jk_\rho = -jk_x x - jk_y y$ in this domain, we obtain the following expressions

$$\tilde{H}_z = -\frac{1}{\omega\mu}[k_\rho \cdot (z \times \tilde{\mathbf{E}}_t) - j\tilde{M}_z] \tag{27}$$

$$\tilde{E}_z = -\frac{1}{\omega\epsilon}[k_\rho \cdot (\tilde{\mathbf{H}}_t \times z) - j\tilde{J}_z] \tag{28}$$

$$-\frac{\partial \tilde{\mathbf{E}}_t}{\partial z} = j\omega\mu \left(\underline{\underline{\mathbf{I}_t}} + \frac{k_\rho k_\rho}{k^2} \right) \cdot (\tilde{\mathbf{H}}_t \times z) + \tilde{\mathbf{M}}_{te} \times z \tag{29}$$

$$-\frac{\partial \tilde{\mathbf{H}}_t}{\partial z} = j\omega\epsilon \left(\underline{\underline{\mathbf{I}_t}} + \frac{k_\rho k_\rho}{k^2} \right) \cdot (z \times \tilde{\mathbf{E}}_t) + z \times \tilde{\mathbf{J}}_{te} \tag{30}$$

$$\tilde{\mathbf{M}}_{te} = \tilde{\mathbf{M}}_t + z \times \frac{k_\rho \tilde{J}_z}{\omega\epsilon} \tag{31}$$

$$\tilde{\mathbf{J}}_{te} = \tilde{\mathbf{J}}_t + z \times \frac{k_\rho \tilde{M}_z}{\omega\mu} \tag{32}$$

where $k_\rho = \sqrt{k_x^2 + k_y^2}$. Let us define a new coordinate system in the transverse plane as in Fig. 2 whose origin coincides with the origin of the xy coordinate system and with the unit vectors $(\hat{u}, \hat{v})$ defined as

$$\begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix} = \begin{pmatrix} \frac{k_x}{k_\rho} & \frac{k_y}{k_\rho} \\ -\frac{k_y}{k_\rho} & \frac{k_x}{k_\rho} \end{pmatrix} \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} \tag{33}$$

We now express both the fields and the sources in this new coordinate system considering their projection both on the $\hat{u}$ and $\hat{v}$ axes, thus obtaining

$$\frac{\partial \tilde{E}_u}{\partial z} = -jk_z \frac{k_z}{\omega\epsilon} \tilde{H}_v + \tilde{M}_{ve} \tag{34}$$

$$\frac{\partial \tilde{H}_v}{\partial z} = -jk_z \frac{\omega\epsilon}{k_z} \tilde{E}_u + \tilde{J}_{ue} \tag{35}$$

$$\frac{\partial \tilde{E}_v}{\partial z} = jk_z \frac{\omega\mu}{k_z} \tilde{H}_u + \tilde{M}_{ue} \tag{36}$$

$$-\frac{\partial \tilde{H}_u}{\partial z} = -jk_z \frac{k_z}{\omega\mu} \tilde{E}_v + \tilde{J}_{ve} \tag{37}$$

Moreover, we can express the current distributions as

$$\tilde{J}_{ue} = \tilde{J}_u \tag{38}$$

$$\tilde{J}_{ve} = \tilde{J}_v + \frac{k_r ho \tilde{M}_z}{\omega\mu} \tag{39}$$

$$\tilde{M}_{ve} = \tilde{M}_v - \frac{k_\rho \tilde{J}_z}{\omega\epsilon} \tag{40}$$

$$\tilde{M}_{ue} = \tilde{M}_u \tag{41}$$

While the longitudinal components yield

$$\tilde{H}_z = -\frac{1}{\omega\mu}[k_\rho(\tilde{E}_v) - j\tilde{M}_z] \tag{42}$$

$$\tilde{E}_z = -\frac{1}{\omega\epsilon}[k_\rho(\tilde{H}_v) - j\tilde{J}_z] \tag{43}$$

We have thus obtained two sets of decoupled equations. The field described by the first and the second equation of (37) represents a field that is TM to z, while the other two expressions represent a field that is TE to z. Thus, the problem can be reduced to two sets of transmission line equations in the form

$$\frac{\partial V}{\partial z} = -jk_z^Q Z^Q I^Q + v^Q \tag{44}$$

$$\frac{\partial I}{\partial z} = -jk_z^Q Y^Q V^Q + i^Q \tag{45}$$

where Q assumes the values of TM,TE. The characteristics of this equivalent transmission line are defined for the TM and TE cases, respectively, as follows

$$V^{TM} = \tilde{E}_u \qquad I^{TM} = \tilde{H}_v \qquad Z^{TM} = \frac{k_z}{\omega\epsilon} \qquad v^{TM} = -\tilde{M}_{ve} \qquad i^{TM} = -\tilde{J}_{ue} \tag{46}$$

$$V^{TE} = \tilde{E}_v \qquad I^{TE} = -\tilde{H}_u \qquad Z^{TE} = \frac{\omega\mu}{k_z} \qquad v^{TE} = -\tilde{M}_{ue} \qquad i^{TE} = -\tilde{J}_{ve} \tag{47}$$

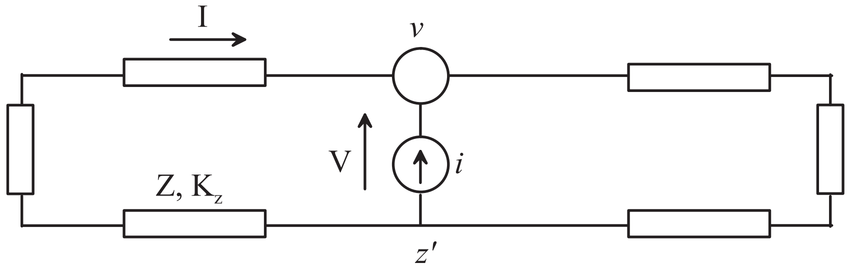

Figure 3: Equivalent transmission line problem

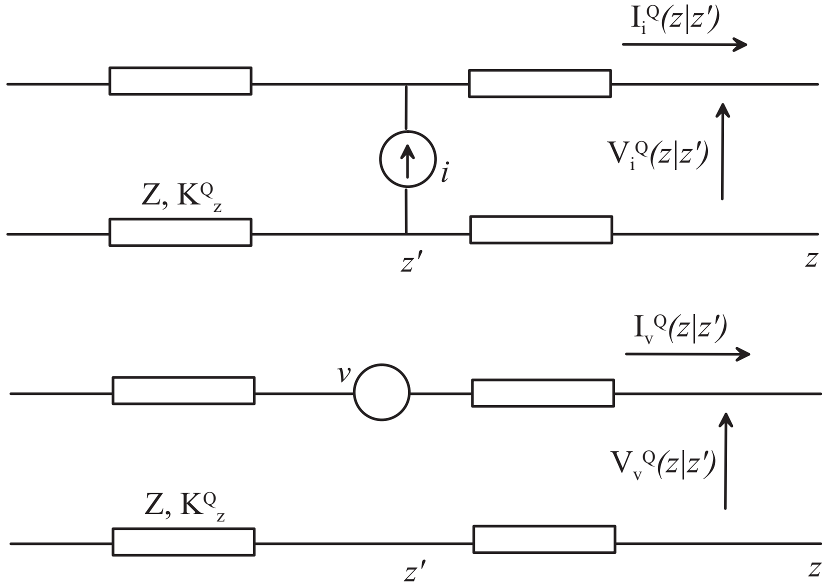

Figure 4: Equivalent problem for the determination of the transmission line Green's functions

We can notice that the original problem, in the spectral domain, reduces to a scalar transmission line problem (see Fig. (3)), which allows us to derive the dyadic Green's function simply by solving the telegraphist's equation. The solution of (47) can be obtained at any point z as the superposition of the solutions of two equivalent transmission line problems as sketched in Fig. 4. For this problem, considering unit-strength impulsive sources, we can write the following equations

$$\frac{\partial V_i^Q}{\partial z} = -jk_z^Q Z^Q I_i^Q \tag{48}$$
$$\frac{\partial I_i^Q}{\partial z} = -jk_z^Q Y^Q V_i^Q + \delta(z - z') \tag{49}$$
$$\frac{\partial V_v^Q}{\partial z} = -jk_z^Q Z^Q I_v^Q + \delta(z - z') \tag{50}$$
$$\frac{\partial I_v^Q}{\partial z} = -jk_z^Q Y^Q V_v^Q \tag{51}$$
$$\tag{52}$$

where $V_v^Q(z|z')$ and $I_v^Q(z|z')$ denote the voltage and the current, respectively, at z due to a unit series voltage source at z', while $V_i^Q(z|z')$ and $I_i^Q(z|z')$ are the voltage and the current, respectively, at z due to a unit shunt current source at z'; and, obviously for the linearity of the problem, $V(z)$ and $I(z)$ are given by the combination of V_v, I_v, V_i, I_i. Upon substituting these equations into (37) and (43), and using (47) we obtain

$$\tilde{E}_u(z) = -V_v^{TM}(z, z')\tilde{M}_{ve}(z') - V_i^{TM}(z, z')\tilde{J}_{ue}(z') \tag{53}$$
$$\tilde{H}_v(z) = -I_v^{TM}(z, z')\tilde{M}_{ve}(z') - I_i^{TM}(z, z')\tilde{J}_{ue}(z') \tag{54}$$
$$\tilde{E}_z(z) = -\frac{1}{\omega\epsilon(z)}\{k_\rho[-I_v^{TM}(z, z')\tilde{M}_{ve}(z') - I_i^{TM}(z, z')\tilde{J}_{ue}(z')] - j\tilde{J}_z(z')\delta(z - z')\} \tag{55}$$
$$\tilde{E}_v(z) = V_v^{TE}(z, z')\tilde{M}_{ue}(z') - V_i^{TE}(z, z')\tilde{J}_{ve}(z') \tag{56}$$
$$\tilde{H}_u(z) = -I_v^{TE}(z, z')\tilde{M}_{ue}(z') + I_i^{TE}(z, z')\tilde{J}_{ve}(z') \tag{57}$$
$$\tilde{H}_z(z) = -\frac{1}{\omega\mu(z)}\{k_\rho[-V_v^{TE}(z, z')\tilde{M}_{ue}(z') - V_i^{TE}(z, z')\tilde{J}_{ve}(z')] - j\tilde{M}_z(z')\delta(z - z')\} \tag{58}$$

In order to achieve our aim of expressing these quantities in the xy coordinate system, we consider the inverse of (33), that is

$$\begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = \begin{pmatrix} \frac{k_x}{k_\rho} & -\frac{k_y}{k_\rho} \\ \frac{k_y}{k_\rho} & \frac{k_x}{k_\rho} \end{pmatrix} \begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix} \tag{59}$$

Applying (59) to $\tilde{J}_{ue}, \tilde{J}_{ve}$ we obtain

$$\begin{pmatrix} \tilde{J}_{ue} \\ \tilde{J}_{ve} \end{pmatrix} = \begin{pmatrix} \frac{k_x}{k_\rho} & \frac{k_y}{k_\rho} & 0 \\ -\frac{k_y}{k_\rho} & \frac{k_x}{k_\rho} & 0 \end{pmatrix} \begin{pmatrix} \tilde{J}_x \\ \tilde{J}_y \\ \tilde{J}_z \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{k_\rho}{\omega\mu(z')} \end{pmatrix} \begin{pmatrix} \tilde{M}_x \\ \tilde{M}_y \\ \tilde{M}_z \end{pmatrix} \tag{60}$$

and, similarly, for $\tilde{M}_{ue}, \tilde{M}_{ve}$ we can write

$$\begin{pmatrix} \tilde{M}_{ue} \\ \tilde{M}_{ve} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{k_\rho}{\omega\epsilon(z\,')} \end{pmatrix} \begin{pmatrix} \tilde{J}_x \\ \tilde{J}_y \\ \tilde{J}_z \end{pmatrix} + \begin{pmatrix} \frac{k_x}{k_\rho} & \frac{k_y}{k_\rho} & 0 \\ -\frac{k_y}{k_\rho} & \frac{k_x}{k_\rho} & 0 \end{pmatrix} \begin{pmatrix} \tilde{M}_x \\ \tilde{M}_y \\ \tilde{M}_z \end{pmatrix} \tag{61}$$

Therefore, applying (59), (60) and (61), it is possible to compute the E and H fields in the xy coordinate system,

$$\begin{pmatrix} \tilde{E}_x \\ \tilde{E}_y \end{pmatrix} = \begin{pmatrix} \frac{k_x}{k_\rho} & -\frac{k_y}{k_\rho} & 0 \\ \frac{k_y}{k_\rho} & \frac{k_x}{k_\rho} & 0 \end{pmatrix} \left[\begin{pmatrix} -\frac{k_x}{k_\rho} V_i^{TM} & -\frac{k_y}{k_\rho} V_i^{TM} & \frac{k_\rho}{\omega\epsilon(z\,')} V_v^{TM} \\ \frac{k_y}{k_\rho} V_i^{TE} & -\frac{k_x}{k_\rho} V_i^{TE} & 0 \end{pmatrix} \begin{pmatrix} \tilde{J}_x \\ \tilde{J}_y \\ \tilde{J}_z \end{pmatrix} + \right.$$
$$\left. \begin{pmatrix} -\frac{k_y}{k_\rho} V_v^{TM} & -\frac{k_x}{k_\rho} V_v^{TM} & 0 \\ \frac{k_x}{k_\rho} V_v^{TE} & \frac{k_y}{k_\rho} V_v^{TE} & -\frac{k_\rho}{\omega\mu(z\,')} V_i^{TE} \end{pmatrix} \begin{pmatrix} \tilde{M}_x \\ \tilde{M}_y \\ \tilde{M}_z \end{pmatrix} \right] \tag{62}$$

$$\begin{pmatrix} \tilde{H}_x \\ \tilde{H}_y \end{pmatrix} = \begin{pmatrix} \frac{k_x}{k_\rho} & -\frac{k_y}{k_\rho} & 0 \\ \frac{k_y}{k_\rho} & \frac{k_x}{k_\rho} & 0 \end{pmatrix} \left[\begin{pmatrix} -\frac{k_y}{k_\rho} I_i^{TE} & \frac{k_x}{k_\rho} I_i^{TE} & 0 \\ -\frac{k_x}{k_\rho} I_i^{TM} & -\frac{k_y}{k_\rho} I_i^{TM} & \frac{k_\rho}{\omega\epsilon(z\,')} I_v^{TM} \end{pmatrix} \begin{pmatrix} \tilde{J}_x \\ \tilde{J}_y \\ \tilde{J}_z \end{pmatrix} + \right.$$
$$\left. \begin{pmatrix} -\frac{k_x}{k_\rho} I_v^{TE} & -\frac{k_y}{k_\rho} I_v^{TE} & \frac{k_\rho}{\omega\mu(z\,')} I_i^{TE} \\ \frac{k_y}{k_\rho} I_v^{TM} & -\frac{k_x}{k_\rho} I_v^{TM} & 0 \end{pmatrix} \begin{pmatrix} \tilde{M}_x \\ \tilde{M}_y \\ \tilde{M}_z \end{pmatrix} \right] \tag{63}$$

In order to obtain the complete expression for the Green's functions in (4), we have to add the longitudinal components E_z, H_z in the xy coordinate system to the previous expressions. For the sake of simplicity, let us consider only the electric current distribution in (62) and (63). Introducing the longitudinal components as well we obtain

$$\begin{pmatrix} \tilde{E}_x \\ \tilde{E}_y \\ \tilde{E}_z \end{pmatrix} = \begin{pmatrix} -\frac{k_x^2 V_i^{TM} + k_y^2 V_i^{TE}}{k_\rho^2} & -\frac{k_y k_x (V_i^{TM} - V_i^{TE})}{k_\rho^2} & \frac{k_x V_v^{TM}}{\omega\epsilon(z\,')} \\ -\frac{k_y k_x (V_i^{TM} - V_i^{TE})}{k_\rho^2} & -\frac{k_y^2 V_i^{TM} + k_x^2 V_i^{TE}}{k_\rho^2} & \frac{k_y V_v^{TM}}{\omega\epsilon(z\,')} \\ \frac{k_x I_i^{TM}}{\omega\epsilon(z\,')} & \frac{k_y I_i^{TM}}{\omega\epsilon(z\,')} & -\frac{1}{\omega\epsilon(z\,')} \left[\frac{k_\rho^2 I_i^{TM}}{\omega\epsilon(z\,')} - j\delta(z - z\,') \right] \end{pmatrix} \begin{pmatrix} \tilde{J}_x \\ \tilde{J}_y \\ \tilde{J}_z \end{pmatrix} \tag{64}$$

$$\begin{pmatrix} \tilde{H}_x \\ \tilde{H}_y \\ \tilde{H}_z \end{pmatrix} = \begin{pmatrix} -\frac{k_y k_x (-I_i^{TE} - I_i^{TM})}{k_\rho^2} & -\frac{k_x^2 I_i^{TE} + k_y^2 I_i^{TM}}{k_\rho^2} & \frac{k_y I_v^{TM}}{\omega\epsilon(z\,')} \\ -\frac{k_y^2 I_i^{TE} - k_x^2 I_i^{TM}}{k_\rho^2} & -\frac{k_y k_x (I_i^{TM} - I_i^{TE})}{k_\rho^2} & -\frac{k_x I_v^{TM}}{\omega\epsilon(z\,')} \\ \frac{k_y V_i^{TE}}{\omega\mu(z\,')} & -\frac{k_x V_i^{TE}}{\omega\mu(z\,')} & 0 \end{pmatrix} \begin{pmatrix} \tilde{J}_x \\ \tilde{J}_y \\ \tilde{J}_z \end{pmatrix} \tag{65}$$

Therefore, the fields due to an electric current distribution in the spectral-domain (i.e., the spectral domain counterpart of (4)) can be written as

$$\tilde{\mathbf{E}} = \int_S \underline{\tilde{\mathbf{G}}}^{Ee} \cdot \tilde{\mathbf{J}} dS \tag{66}$$

$$\tilde{\mathbf{H}} = \int_S \underline{\tilde{\mathbf{G}}}^{He} \cdot \tilde{\mathbf{J}} dS \tag{67}$$

where $\underline{\tilde{\mathbf{G}}}^{He}$ and $\underline{\tilde{\mathbf{G}}}^{Ee}$ are given by the elements of the matrix in (65) and (64), respectively. The aim is to evaluate the dyadic components of the Green's function in (11). Considering (67) and the field expression in terms of scalar and vector potentials, which are rewritten here for the sake of simplicity without magnetic current distributions

$$\mathbf{E} = -j\omega \mathbf{A} - \nabla \Phi \tag{68}$$

$$\mu \mathbf{H} = \nabla \times \mathbf{A} \tag{69}$$

$$\mathbf{A} = \mu \int \underline{\mathbf{G}}_A \cdot J dS \tag{70}$$

it follows that, considering the counterparts in the spectral-domain of all the quantities, we obtain

$$\mu \cdot \underline{\tilde{\mathbf{G}}}^{He} = \nabla \times \underline{\tilde{\mathbf{G}}}_A \tag{71}$$

In a matrix form, considering the expression for the ∇ operator in the spectral domain and (11), we obtain

$$\mu \begin{pmatrix} -\frac{k_y k_x (-I_i^{TE} - I_i^{TM})}{k_\rho^2} & -\frac{k_x^2 I_i^{TE} + k_y^2 I_i^{TM}}{k_\rho^2} & \frac{k_y I_v^{TM}}{\omega \epsilon (z\,')} \\ -\frac{k_y^2 I_i^{TE} - k_x^2 I_i^{TM}}{k_\rho^2} & -\frac{k_y k_x (I_i^{TM} - I_i^{TE})}{k_\rho^2} & -\frac{k_x I_v^{TM}}{\omega \epsilon (z\,')} \\ \frac{k_y V_i^{TE}}{\omega \mu (z\,')} & -\frac{k_x V_i^{TE}}{\omega \mu (z\,')} & 0 \end{pmatrix} =$$

$$\begin{pmatrix} -jk_y \tilde{G}_{zx}^{mi} & -jk_y \tilde{G}_{zy}^{mi} - \frac{\partial \tilde{G}_{xx}^{mi}}{\partial z} & -jk_y \tilde{G}_{zz}^{mi} \\ jk_x \tilde{G}_{zx}^{mi} + \frac{\partial \tilde{G}_{xx}^{mi}}{\partial z} & jk_x \tilde{G}_{zy}^{mi} & jk_x \tilde{G}_{zz}^{mi} \\ jk_y \tilde{G}_{xx}^{mi} & -jk_x \tilde{G}_{xx}^{mi} & \end{pmatrix} \tag{72}$$

This allows us to find each component of $\underline{\tilde{\mathbf{G}}}_A$ in (11)

$$-jk_x \tilde{G}_{xx}^{mi} = \frac{\mu}{\omega} k_x V_i^{TE} \rightarrow G_{xx}^{mi} = \frac{1}{j\omega} V_i^{TE} \tag{73}$$

$$jk_x \tilde{G}_{zz}^{mi} = -\frac{\mu}{\omega \epsilon} k_x I_v^{TM} \rightarrow G_{zz}^{mi} = -\frac{\mu}{j\omega \epsilon} I_v^{TM} \tag{74}$$

$$-jk_x \tilde{G}_{zy}^{mi} = \frac{\mu}{k_\rho^2} k_y k_x (I_i^{TE} - I_i^{TM}) \rightarrow G_{zy}^{mi} = -j \frac{\mu}{k_\rho^2} k_y (I_i^{TE} - I_i^{TM}) \tag{75}$$

$$-jk_y \tilde{G}_{zx}^{mi} = \frac{\mu}{k_\rho^2} k_x k_y (-I_i^{TE} + I_i^{TM}) \rightarrow G_{zx}^{mi} = -\frac{\mu}{k_\rho^2} k_x (-I_i^{TE} + I_i^{TM}) \tag{76}$$

Following an equivalent procedure for the expression of $\underline{\tilde{\mathbf{G}}}_A$ in (12), we are able to derive an analogous relation for the other components.

Mixed potential EFIE formulations

In paragraph it has been pointed out that the choice of K_Φ^{mi} and $\mathbf{P}^{mi}$ is not unique. Based on a different choice of these parameters, different mixed potential formulations can be derived [3]. In this section we briefly present three of these formulations in order to explain our choice. As already mentioned, the dyadic kernel $\underline{\mathbf{K}}_A^{mi}$ can be expressed as follows

$$\underline{\mathbf{K}}_A^{mi}(\mathbf{r}|\mathbf{r}\,') = \underline{\mathbf{G}}_A^{mi}(\mathbf{r}|\mathbf{r}\,') + \nabla\mathbf{P}^{mi}(\mathbf{r}|\mathbf{r}\,'). \tag{77}$$

where $\mathbf{P}^{mi}$ is the "correction factor". Unfortunately, this term introduces new elements into the dyadic kernel. So we have to operate a choice in order to reduce as much as possible, the number of non-zero dyadic elements. For the vector's Green's function in (11) or (12) we can see that the x and y component of $\mathbf{P}^{mi}$ are not independent. This leaves us only two degrees of freedom for each buG_A^{mi}, hence

$$P_x^{mi} = P_y^{mi} = 0\,, P_z^{mi} \neq 0 \tag{78}$$

$$P_x^{mi} \neq 0 P_y^{mi} \neq 0\,, P_z^{mi} = 0 \tag{79}$$

We can write each component of (77) in the spectral domain, substituting the following to the vector's Green's function in (11) and (12), respectively

$$\frac{j\omega}{k_m^2}\left(-jk_x\tilde{G}_{xx}^{mi} + \frac{\partial\tilde{G}_{zx}^{mi}}{\partial z}\right) = \frac{1}{j\omega}jk_x\tilde{K}_\phi^{mi} + j\omega\tilde{P}_x^{mi} \tag{80}$$

$$\frac{j\omega}{k_m^2}\left(-jk_y\tilde{G}_{xx}^{mi} + \frac{\partial\tilde{G}_{zy}^{mi}}{\partial z}\right) = \frac{1}{j\omega}jk_y\tilde{K}_\phi^{mi} + j\omega\tilde{P}_y^{mi} \tag{81}$$

$$\frac{j\omega}{k_m^2}\frac{\partial\tilde{G}_{zz}^{mi}}{\partial z} = \frac{1}{j\omega}\frac{\partial\tilde{K}_\phi^{mi}}{\partial z\,'} + j\omega\tilde{P}_z^{mi} \tag{82}$$

$$\frac{j\omega}{k_m^2}\left(-jk_x\tilde{G}_{xx}^{mi} - jk_y\tilde{G}_{xy}^{mi}\right) = \frac{1}{j\omega}jk_x\tilde{K}_\phi^{mi} + j\omega\tilde{P}_x^{mi} \tag{83}$$

$$\frac{j\omega}{k_m^2}\left(-jk_x\tilde{G}_{xy}^{mi} - jk_y\tilde{G}_{yy}^{mi}\right) = \frac{1}{j\omega}jk_y\tilde{K}_\phi^{mi} + j\omega\tilde{P}_y^{mi} \tag{84}$$

$$\frac{j\omega}{k_m^2}\frac{\partial\tilde{G}_{zz}^{mi}}{\partial z} = \frac{1}{j\omega}\frac{\partial\tilde{K}_\phi^{mi}}{\partial z\,'} + j\omega\tilde{P}_z^{mi} \tag{85}$$

Formulation A

Let us consider the dyadic Green's function given in (85) [3]. In this formulation we force P_x^{mi},and P_y^{mi} to be zero; therefore, considering the expressions carried out from the transmission line model, K_Φ can be interpreted as the scalar potential of a point charge associated with a horizontal dipole. Solving (85) with $P_x^{mi} = P_y^{mi} = 0$ and using the counterpart of (76) for (12) we obtain the expressions of $K_\Phi, \tilde{P}_z^{mi}$. Substituting these in (17) we obtain the relation between the dyadic kernel components and the equivalent transmission line parameters, from which we can deduce that $\tilde{\underline{\mathbf{K}}}_A = \tilde{\underline{\mathbf{G}}}_A$, when the object is confined in a single layer (m=i), otherwise we introduce two new terms into the dyadic expression. The main advantage of this formulation is the cancellation of the contour integrals in (20), thanks to the continuity of the scalar potential kernel at the $i-th$ interface, with respect to the $z\,'$ coordinate. There is no continuity with respect to the z coordinate.

Formulation B

Within this formulation as well we use (12), but now we choose $P_z^{mi} = 0$ [9]. Based on the previous results, this time K_Φ can be interpreted as the scalar potential of a point charge associated with a vertical dipole. Following a procedure similar to that described for the A formulation, we deduce that also in this case $\tilde{\underline{\mathbf{K}}}_A = \tilde{\underline{\mathbf{G}}}_A$, when the object is confined in a single layer (m=i), otherwise we introduce two new terms into the dyadic expression. The main drawback of this formulation is that we do not obtain the cancellation of the contour integrals in the expression of Φ^{mi} since the continuity of the scalar potential kernel at the $i-th$ interface corresponds to the z coordinate and not to the $z\,'$ coordinate.

Formulation C

In order to obtain this formulation we use (11) and choose $P_x^{mi} = P_y^{mi} = 0$. K_Φ can be now interpreted as the scalar potential of a point charge associated with a horizontal dipole. Main advantage: cancellation of the contour integrals in (20) again thanks to the continuity of the scalar potential kernel at the $i-th$ interface with respect to the $z\,'$ coordinate as in Formulation A; but in this case we also have the continuity of the z coordinate. The main drawback is the introduction of additional terms to the dyadic kernel even when the object is confined to a single layer. Why can formulation C be preferable to the others? It is preferable with objects penetrating an interface because it does not have any contour integrals and because its scalar potential kernel is continuous at the interfaces with respect to both z and $z\,'$, which results in a simplification when we study general structures in which we can have objects not confined to a single layer. The drawback of this formulation is that we obtain a more complex equation compared to those in formulation A, so it is convenient to use it only when both vertical and horizontal currents are present. In the case of sources confined in a single layer or when only a type of exitation is present (i.e. vertical or horizontal) the most convenient approach is to use of the results of formulation A.

Closed form of the Green's function for multilayered structures

A closed form of the Green's function in the spatial domain is one of the possible means for reducing the convergence problems and computational efforts when we apply a method of moments. In fact, if we express the spatial domain Green's function in a closed form, the inner products in a MoM matrix formulation become two-dimensional integrals over finite ranges, and the time-consuming part of the method of moments in the spatial domain, which entails the evaluation of the integral representations of the Green's functions, is completely avoided. It is also a well known fact that a closed form of the Green's function in planar layered media can be obtained analytically only in the spectral domain. Therefore, a closed form of the Green's function in the spatial domain can be achieved only by applying some approximation techniques. In the following sections we derive a possible approximation.

Spectral domain Green's function

Let us consider a general multilayered environment, as depicted in Fig. 5. It is assumed that all layers, including the ground plane, extend to infinity in the horizontal plane, and that the conductors are lossless and infinitesimally thin. We denote by ϵ_i, μ_i and d_i the permittivity, permeability and thickness of the layer i. The spectral domain Green's functions in the source layer in the cases of the Horizontal Electric Diplole (HED), Vertical Electric Diplole (VED), Horizontal Magnetic Diplole (HMD) and Vertical Magnetic Diplole (VMD) sources are listed in the following paragraphs. These are directly obtained from (76) once the pertinent expressions for voltage, current and the characteristic impedance of the equivalent transmission line being tested have been substituted. We note that the $z|z'$ dependence of the fields in the source region can be written as the sum of the direct term and the up and down-going waves due to the reflections from the top and bottom bound-

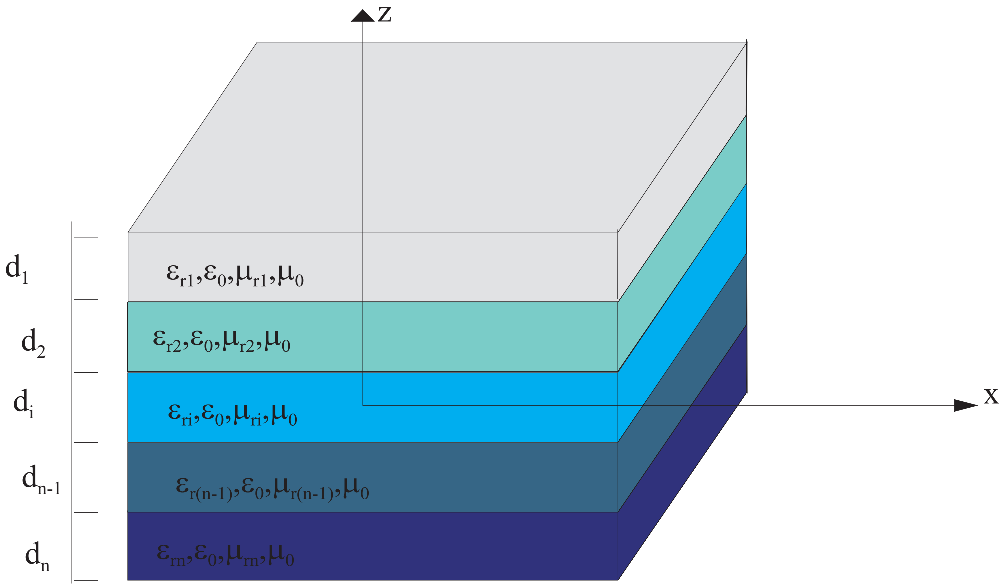

Figure 5: Generic multilayered medium geometry.

aries. The coefficients of the up and down-going waves can be obtained in terms of generalized reflection coefficients by applying the appropriate boundary conditions [21].

HED

$$\widetilde{G}^A_{xx} = \frac{\mu_i}{2jk_z}\left[e^{-jk_z|z-z'|} + A^e_h e^{jk_z(z-z')} + C^e_h e^{-jk_z(z-z')}\right] , \tag{86}$$

$$\frac{\widetilde{G}^A_{zx}}{k_x} = \frac{-\mu_i}{2jk_\rho^2}\left[(A^e_h + B^e_h)e^{jk_z(z-z')} + (D^e_h - C^e_h)e^{-jk_z(z-z')}\right] , \tag{87}$$

$$\widetilde{G}^q_x = \frac{1}{2j\epsilon_i k_z}\left[e^{-jk_z|z-z'|} + \frac{k_i^2 A^e_h + k_z^2 B^e_h}{k_\rho^2} e^{jk_z(z-z')} + \frac{k_i^2 C^e_h - k_z^2 D^e_h}{k_\rho^2} e^{-jk_z(z-z')}\right] \tag{88}$$

VED

$$\widetilde{G}^A_{zz} = \frac{\mu_i}{2jk_z}\left[e^{-jk_z|z-z'|} + A^e_v e^{jk_z(z-z')} + B^e_v e^{-jk_z(z-z')}\right] , \tag{89}$$

$$\widetilde{G}^q_z = \frac{1}{2j\epsilon_i k_z}\left[e^{-jk_z|z-z'|} + C^e_v e^{jk_z(z-z')} + D^e_v e^{-jk_z(z-z')}\right] , \tag{90}$$

HMD

$$\widetilde{G}^F_{xx} = \frac{\epsilon_i}{2jk_z}\left[e^{-jk_z|z-z'|} + A^m_h e^{jk_z(z-z')} + C^m_h e^{-jk_z(z-z')}\right] , \tag{91}$$

$$\frac{\widetilde{G}^F_{zx}}{k_x} = \frac{-\epsilon_i}{2jk_\rho^2}\left[(A^m_h + B^m_h)e^{jk_z(z-z')} + (D^m_h - C^m_h)e^{-jk_z(z-z')}\right] , \tag{92}$$

$$\widetilde{G}^q_x = \frac{1}{2j\mu_i k_z}\left[e^{-jk_z|z-z'|} + \frac{k_i^2 A^m_h + k_z^2 B^m_h}{k_\rho^2} e^{jk_z(z-z')} + \frac{k_i^2 C^m_h - k_z^2 D^m_h}{k_\rho^2} e^{-jk_z(z-z')}\right] \tag{93}$$

VMD

$$\widetilde{G}^F_{zz} = \frac{\epsilon_i}{2jk_z}\left[e^{-jk_z|z-z'|} + A^m_v e^{jk_z(z-z')} + B^m_v e^{-jk_z(z-z')}\right] , \tag{94}$$

$$\widetilde{G}^q_z = \frac{1}{2j\mu_i k_z}\left[e^{-jk_z|z-z'|} + C^m_v e^{jk_z(z-z')} + D^m_v e^{-jk_z(z-z')}\right] , \tag{95}$$

The coefficients $A^{e,m}_{h,v}, B^{e,m}_{h,v}, C^{e,m}_{h,v}, D^{e,m}_{h,v}$ are functions of the generalized reflection coefficients $\widetilde{R}_{TE,TM}$ and are given by

$$A^{e,m}_h = \left[e^{-2jk_{zi}(d_i - z')} + \tilde{R}^{i,i-1}_{TE,TM} e^{-2jk_{zi}d_i}\right] \tilde{R}^{i,i+1}_{TE,TM} M^i_{TE,TM} , \tag{96}$$

$$B^{e,m}_h = \left[e^{-2jk_{zi}(d_i - z')} - \tilde{R}^{i,i-1}_{TM,TE} e^{-2jk_{zi}d_i}\right] \tilde{R}^{i,i+1}_{TM,TE} M^i_{TM,TE} , \tag{97}$$

$$C^{e,m}_h = \left[e^{-2jk_{zi}(d_i + z')} + \tilde{R}^{i,i+1}_{TE,TM} e^{-2jk_{zi}d_i}\right] \tilde{R}^{i,i-1}_{TE,TM} M^i_{TE,TM} , \tag{98}$$

$$D^{e,m}_h = \left[-e^{-2jk_{zi}(d_i + z')} + \tilde{R}^{i,i+1}_{TM,TE} e^{-2jk_{zi}d_i}\right] \tilde{R}^{i,i-1}_{TM,TE} M^i_{TM,TE} , \tag{99}$$

$$A^{e,m}_v = \left[e^{-2jk_{zi}(d_i + z')} + \tilde{R}^{i,i+1}_{TE,TM} e^{-2jk_{zi}d_i}\right] \tilde{R}^{i,i+1}_{TE,TM} M^i_{TE,TM} , \tag{100}$$

$$B^{e,m}_v = \left[e^{-2jk_{zi}(d_i - z')} - \tilde{R}^{i,i-1}_{TM,TE} e^{-2jk_{zi}d_i}\right] \tilde{R}^{i,i+1}_{TM,TE} M^i_{TM,TE} , \tag{101}$$

$$C^{e,m}_v = \left[-e^{-2jk_{zi}(d_i + z')} + \tilde{R}^{i,i+1}_{TE,TM} e^{-2jk_{zi}d_i}\right] \tilde{R}^{i,i-1}_{TE,TM} M^i_{TE,TM} , \tag{102}$$

$$D^{e,m}_v = \left[-e^{-2jk_{zi}(d_i - z')} + \tilde{R}^{i,i+1}_{TM,TE} e^{-2jk_{zi}d_i}\right] \tilde{R}^{i,i-1}_{TM,TE} M^i_{TM,TE} , \tag{103}$$

Considering the equivalent transmission line, we can derive the generalized reflection coefficients, $\tilde{R}^{i,i+1}_{TE}, \tilde{R}^{i,i-1}_{TE}, \tilde{R}^{i,i-1}_{TM}, \tilde{R}^{i,i+1}_{TM}$, as functions of the Fresnel reflection coefficients. These are given by[21]:

$$\tilde{R}_{i,j} = \frac{r_{i,j} + \tilde{R}_{i+1,j+1} e^{-2jk_{zj}d_j}}{1 + r_{i,j}\tilde{R}_{i+1,j+1} e^{-2jk_{zj}d_j}} \tag{104}$$

where M_{TE}, M_{TM} can be defined as

$$M_{TE}(i) = [1 - \tilde{R}_{TE}^{i,i-1} \tilde{R}_{TE}^{i,i+1} e^{-2jk_{zi}d_i}]^{-1} \tag{105}$$

$$M_{TM}(i) = [1 - \tilde{R}_{TM}^{i,i-1} \tilde{R}_{TM}^{i,i+1} e^{-2jk_{zi}d_i}]^{-1} \tag{106}$$

Approximation of the Green's function

Let us consider again the multilayered structure in Fig. 5. We suppose a lossless dielectric. The origin of the coordinate system we have used is located at the bottom of the source layer. The spatial domain Green's function is represented by the 2-D inverse Fourier Transform, better known as the Sommerfeld Integral [22],

$$G^{A,q} = \frac{1}{4\pi} \int_{SIP} k_\rho H_0^{(2)}(k_\rho \rho) \underline{\widetilde{\mathbf{G}}}^{A,q} , \tag{107}$$

where $H_0^{(2)}(k_\rho \rho)$ is the Hankel function of the second kind, SIP is the Sommerfeld Integration Path (sketched in Fig. 6) and $\widetilde{G}^{A,q}$ is the spectral domain Green's function. Apart from special cases, the Sommerfeld integral in eq. (107), cannot be evaluated analytically. It is worth noting that the Sommerfeld Integral (107) can be evaluated analytically by using the well-known Sommerfeld identity (108) when the Green's function is approximated by complex exponentials [?]

$$\frac{e^{-jkR}}{R} = \frac{-j}{2} \int_{SIP} dk_\rho k_\rho H_0^{(2)}(k_\rho \rho) \frac{e^{-jk_z|z|}}{k_z} . \tag{108}$$

Therefore, after computing the spectral domain Green's function, we can approximate it through a set of complex exponentials, whose complex coefficients are extracted by using the Generalized Pencil of Function method [10]. Before applying this procedure, we have to deform the integration path in order for the Green's function to be in a form suitable for this approach. Once we have expressed the Green's function in this approximated form, we can easily apply the Sommerfeld identity. According to the results described in section we can write the terms of the Green's function as the sum of three different contributions, the direct wave, the up-going and down-going terms, which depend on z and $-z$, respectively.

$$\widetilde{F}(z, z') = e^{jk_{zm}|z-z'|} + A_m e^{-jk_{mz}z} + B_m e^{-jk_{mz}z} , \tag{109}$$

where A_m, B_m depend on z' and are directly related to the specific values at the interfaces. Following an approach similar to those used in [17] and in [19] we can summarize the entire procedure as follows:

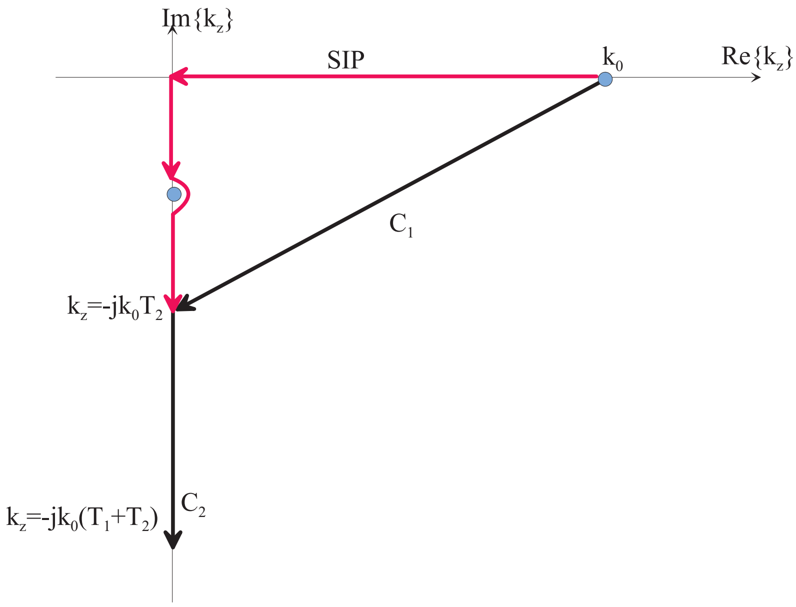

Figure 6: Two levels GPOF sampling paths.

- the complete set of components for the Green's functions for the scalar and vector potentials are computed in the spectral domain;
- these functions are sampled along a path in the k_ρ plane and then approximated by using the $GPOF$ method.

Based on the approach proposed in [17], we use a two-step approximation, so the Green's function in the spatial domain is approximated through two sets of complex exponentials. Since we have not assumed the extraction of the surface wave poles, we need an accurate approximation for the Green's function in both the near field and the far field. This cannot be easily done if we apply a single step approximation procedure, due to the influence of the bounds of the modified path on the accuracy of the approximation. A higher limit value would give a better approximation in the far field, while with a lower one we will obtain a better result in the near field [23]. Therefore, in order to achieve a good result along the chosen test-source range, we choose two different paths C_1, C_2 along which the Green's function is sampled. The chosen paths are sketched in Fig. 6 and are given by the equations (110),(111):

$$C_1 : k_{z1} = -jk_0 \left[T_2 + t\right] \text{ with } 0 \leq t \leq T_1 \tag{110}$$

$$C_2 : K_{z2} = k_0 \left[-jt + \left(1 - \frac{t}{T_2}\right)\right] \text{ with } 0 \leq t \leq T_2 \tag{111}$$

where T_1, T_2 are the bounds of the sampling interval. The choice of T_1 and T_2 is a crucial step for the approximation and depends on the characteristics of the structure itself. Once we have set these bounds, we sample the Green's function along the path C_1 and we approximate this set of samples with the GPOF method obtaining

$$\widetilde{\underline{\mathbf{G}}} \simeq \sum_{n=1}^{N_1} a_{1n} e^{-\alpha_{1n} k_{z1}} \,. \tag{112}$$

where a_{1n}, α_{1n} are the complex coefficients resulting from the GPOF procedure.
- We suppose that this series, obtained as an approximation of the Green's function along C_1, is valid throughout the whole domain. Then we evaluate the Green's function along the C_2 path and compute the error function as the difference between this function and the previous one. The resulting "error function" is approximated through the GPOF method.
Following this procedure we can write the entire spectral domain Green's function as

$$\widetilde{\underline{\mathbf{G}}}^{A,q} = \sum_{n=1}^{N_1} a_{1n} e^{-\alpha_{1n} k_z} + \sum_{m=1}^{N_2} a_{2m} e^{-\alpha_{2m} k_z} \,, \tag{113}$$

where $a_{1n}, \alpha_{1n}, a_{2n}, \alpha_{2n}$ are the complex coefficents obtained with the GPOF at the first and second step, respectively. N_1, N_2 are the number of exponentials necessary to approximate the pertinent function. Substituting (113) in (107) we obtain

$$\underline{\mathbf{G}}^{A,q} = \frac{1}{4\pi} \int_{C_1+C_2} dK_\rho k_\rho H_0^{(2)}(k_\rho \rho) \sum_{n=1}^{N_1} a_{1n} e^{-\alpha_{1n} k_z} + \frac{1}{4\pi} \int_{C_2} dK_\rho k_\rho H_0^{(2)}(k_\rho \rho) \sum_{m=1}^{N_2} a_{2m} e^{-\alpha_{2m} k_z} \,, \tag{114}$$

It is worth noting that this choice leads to an analytical evaluation of the spatial domain Green's function through the Sommerfeld Identity. Considering eqs. (114) and (108) we obtain

$$\underline{\mathbf{G}}^{A,q} = \frac{1}{4\pi} \sum_{n=1}^{N_1} a_{1n} \frac{e^{-jkR_{1n}}}{R_{1n}} + \frac{1}{4\pi} \sum_{m=1}^{N_2} a_{2m} \frac{e^{-jkR_{2m}}}{R_{2m}} \,, \tag{115}$$

where $R_{1n} = \sqrt{|\rho - \rho\,'|^2 + (j\alpha_{1n} \pm (z \pm z')^2}$, and $R_{2m} = \sqrt{|\rho - \rho\,'|^2 + (j\alpha_{2m} \pm (z \pm z')^2}$.

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