ANALYSIS OF STAND-ALONE RECTANGULAR PATCH ANTENNAS BY THE TRANSMISSION LINE MODEL

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The rectangular patch is by far the most widely used microstrip antenna configuration. It is very easy to analyze by using both the transmission line and cavity models, which are most accurate for thin substrates [1].

A rectangular microstrip radiating element excited in the lowest resonant mode is schematically shown in Figure 1. The element is fed by a transmission line in the plane of the patch, or from the back by a coaxial cable whose inner conductor extends through the dielectric and is soldered to the radiating patch, to give a field distribution which is usually uniform along the width *w*. In this case the structure can be treated as a microstrip line resonator which is open circuited at both ends and supports quasi-TEM modes. Furthermore, radiation essentially takes place at the ends of the line resonators, while radiation from the strip is negligible for wide strips. Accordingly, the simplest analytical description of a rectangular microstrip patch exploits transmission-line theory and models the patch as two uniformly illuminated radiating slots, taking into account the fringing of the fields at the patch edges, separated by a transmission line of very low characteristic impedance Z_c [3], as shown in Figure 2. The length of this line ℓ is about half a wavelength to reverse the field in the slots. Indeed, the fields at the edges can be resolved into both normal and tangential components with respect to the ground plane. However, the normal components of the electric field are in opposite directions, and their contributions tend to cancel out each other in the broadside direction. Contrarily, the components of the fields parallel to the ground plane adds in phase to give a maximum radiated field normal to the surface of the structure.

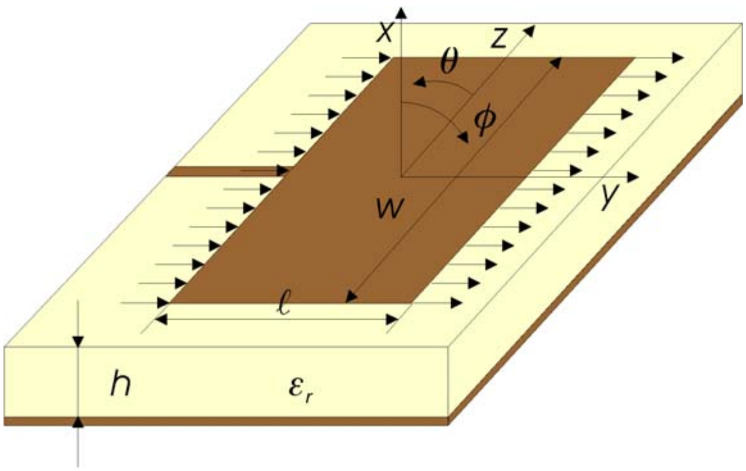

Figure 1. Rectangular microstrip radiating element.

Input impedance

Each radiating slot can be represented by a parallel equivalent admittance Y_s with conductance G_s and susceptance B_s (Figure 2). The slots are denoted by #1 and #2. The equivalent admittance of slot #1, based on the results for an infinitely long uniform narrow slot radiating into a half-space [4], is given by:

$$G_{s1} + jB_{s1} \cong \frac{\pi w}{\lambda_0 Z_0}\left\{1 + j\left[1 - 0.636\ln\left(k_0 h\right)\right]\right\}, \quad \frac{h}{\lambda_0} \ll 1, \tag{1.1}$$

where λ_0, Z_0, and k_0 are the free-space wavelength, intrinsic impedance, and wavenumber, respectively, and the slot width is assumed approximately equal to the substrate thickness h. Since the slots are identical (except for fringing effects potentially associated with the feed point on one of the edge), an identical expression holds for the admittance of slot #2. Assuming no field variation along the direction parallel to the radiating edges, and neglecting mutual coupling between the slots, the input admittance of the rectangular patch can be found by transforming one of the slots through the low impedance line to give:

$$Y_{in} = G_{s1} + jB_{s1} + Y_c \frac{G_{s1} + j\left[B_{s1} + Y_c \tan\left(k_d \ell\right)\right]}{Y_c - B_{s1}\tan\left(k_d \ell\right) + jG_{s1}\tan\left(k_d \ell\right)}, \tag{1.2}$$

where k_d and Y_c are the propagation constant and the characteristic impedance of the microstrip transmission line modelling the patch.

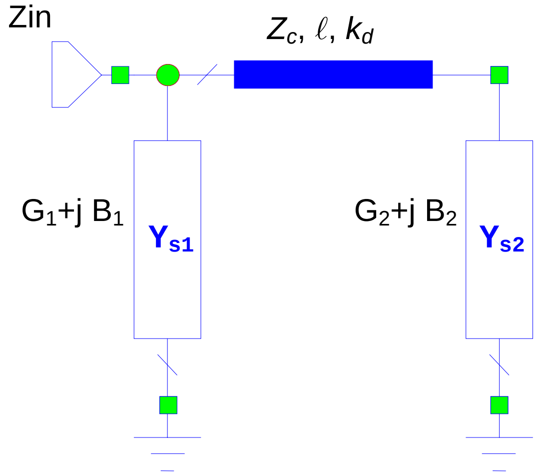

Figure 2. Equivalent network of a microstrip radiating element.

By properly choosing the length ℓ of the radiating element, the admittance of slot #2 after transformation becomes

$$\widetilde{G}_{s2} + j\widetilde{B}_{s2} = G_{s1} - jB_{s1}, \tag{1.3}$$

so that the total input admittance at resonance simply reads as [5]

$$Y_{in} = 2G_{s1}. \tag{1.4}$$

In particular, this occurs when the length of the line is such that:

$$\tan\left(k_d \ell\right) = \frac{2Y_c B_{s1}}{G_{s1}^2 + B_{s1}^2 - Y_c^2}, \tag{1.5}$$

which for practical values of the parameters yields a resonant length of the patch slightly shorter than half a wavelength. This reduction is due to the fringing of the fields at the radiating edges which makes the electrical length of the patch appear greater than its physical dimensions. An initial approximation for the patch length can be ℓ = 0.48 λ_d to 0.49 λ_d, where the guided wavelength λ_d depends on the effective dielectric constant ε_{reff} of the inhomogeneous microstrip line: $\lambda_d = \lambda_0 \sqrt{\varepsilon_{reff}}$. The value of ε_{reff} is slightly less then ε_r because the fringing fields around the periphery of the patch are not confined in the dielectric substrate but are also spread in air. The effective permittivity can be determined in the low-frequency limit as [6]:

$$\varepsilon_{reff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left(1 + 10\frac{h}{w}\right)^{-1/2}, \quad \frac{w}{h} > 1. \tag{1.6}$$

Once the effective permittivity is known, the characteristic impedance Z_c of the microstrip line corresponding to the patch is given by [7]

$$Z_c = \frac{120\pi}{\sqrt{\varepsilon_{reff}}}\left[\frac{w}{h} + 1.393 + 0.667\ln\left(\frac{w}{h} + 1.444\right)\right]^{-1}, \quad \frac{w}{h} > 1. \tag{1.7}$$

Moreover, as a function of the effective dielectric constant and the width to height ratio (*w*/*h*), the distance by which the patch length should be extended because of the fringing effects can be more accurately estimated - a more accurate approximation can be derived for the effective patch length. The open-end extension $\Delta\ell$ of the patch considered as a semi-infinite microstrip line of width *w*. The most popular approximate relation for the normalized extension of the line is [8]

$$\frac{\Delta\ell}{h} = 0.412\frac{\left(\varepsilon_{reff} + 0.3\right)\left(\frac{w}{h} + 0.264\right)}{\left(\varepsilon_{reff} - 0.258\right)\left(\frac{w}{h} + 0.8\right)}. \tag{1.8}$$

Improved expressions for this line extension were more recently given in [9] and [10]. Since the length of the patch is to be extended by $\Delta\ell$ on each side, the effective length of the patch to be considered in the design is:

$$\ell_{eff} = \ell + 2\Delta\ell. \tag{1.9}$$

By using the network model in Figure 2, the input impedance at different sections on the patch can be also easily estimated. This could help choosing the proper location for the feed to realize the best impedance matching with the generator. Indeed, the input impedance of a patch at its edges is usually too high for direct connection to the feeding line, whose standard impedance is 50 Ohm. A matching network (e.g. a quarter wavelength transformer) can be designed to achieve a satisfactory return loss at the resonance frequency. Alternatively, it is observed that smaller radiation resistance can be obtained by displacing the feed point towards the centre of the antenna, as it is shown in the following. In view of this, common feeding techniques are the probe feed, which has the advantage that the inner conductor of the coaxial connector extending through the dielectric and soldered to the patch can be placed at any desired location inside it, and inset feed, which consists in moving offset from the edge the feed line-patch transition [12].

At resonance, the input impedance at an arbitrary feed point, at a distance *x* from one end of the microstrip element is purely real. By transforming the slot admittances to the common point and adding them together, the input impedance at resonance is found as:

$$Z_{in}(x)=\frac{1}{2G_{s1}}\left[\cos^2\left(\beta_c x\right)+\frac{G_{s1}^2+B_{s1}^2}{Y_c^2}\sin^2\left(\beta_c x\right)-\frac{B_{s1}}{Y_c}\sin\left(2\beta_c x\right)\right], \tag{1.10}$$

When x=0 this expression simply reduces to (1.4). Usually $G_{s1}/Y_c \ll 1$ and $B_{s1}/Y_c \ll 1$, therefore (1.10) simplifies to

$$Z_{in}(x)=\frac{1}{2G_{s1}}\cos^2\left(\beta_c x\right). \tag{1.11}$$

This expression allows one to approximately determine the required inset feed distance or the location of the probe feed inside the patch to match its input impedance.

On the basis of the simplified formulation that has been described, a design procedure will assume that the specified information includes the dielectric constant of the substrate ε_r, the resonant frequency f_0, and the height of the substrate h.

As a first step the width of the patch is to be determined.

In principle, as it appears from (1.4) and (1.1), one could choose the patch width to obtain a desired input resonant resistance, so as to easily match the patch to the voltage generator. However, the width w is usually chosen such that it lies in the range $\ell < w < 2\ell$ to have good radiation characteristics; in fact, if w is too large, higher order modes will move closer to the design frequency. Specifically, for an efficient radiator, a practical width that leads to good radiation efficiency is [11]:

$$w=\frac{c}{2f_0}\sqrt{\frac{2}{\varepsilon_r+1}}, \tag{1.12}$$

where c is the free-space velocity of light.

Once w is found, the extension of the patch length can be calculated through (1.8). The actual length of the patch follows from imposing $\ell_{eff}=\lambda_d/2$, that is

$$\ell=\frac{c}{2f_0\sqrt{\varepsilon_{eff}}}-2\Delta\ell. \tag{1.13}$$

Finally, the appropriate location for the feed to realize a good impedance matching with the generator can be determined with the help of (1.11).

Radiation pattern

The total field radiated by a rectangular patch can be seen as the sum of a two-element array with each element representing one of the slots. Since the slots are identical, this is accomplished by using an array factor for the two slots.

Each slot will produce exactly the same field as a uniform magnetic current of length w radiating into free space. The far-field approximation will only give a ϕ-directed component of the radiated electric field:

$$E_\phi = j\frac{k_0 hwE_0 e^{-jk_0 r}}{2\pi r}\sin\theta\frac{\sin\left(\frac{k_0 h}{2}\sin\theta\cos\phi\right)}{\frac{k_0 h}{2}\sin\theta\cos\phi}\frac{\sin\left(\frac{k_0 w}{2}\cos\theta\right)}{\frac{k_0 w}{2}\cos\theta}. \tag{1.14}$$

For very small heights ($k_0 h \ll 1$), (1.14) reduces to

$$E_\phi = j\frac{V_0 e^{-jk_0 r}}{\pi r}\sin\theta\frac{\sin\left(\frac{k_0 w}{2}\cos\theta\right)}{\cos\theta}, \tag{1.15}$$

where $V_0 = hE_0$ is the voltage across the slot.

According to array theory [13], the array factor for the two elements, of the same magnitude and phase, separated by a distance ℓ_{eff} along the *y* direction, is:

$$AF = 2\cos\left(\frac{k_0 \ell_{eff}}{2}\sin\theta\sin\phi\right), \tag{1.16}$$

where ℓ_{eff} is the effective length of (1.9). Thus, the total electric field for the two slots, that is for the microstrip antenna, is simply given by the multiplication of (1.14) and (1.16) as

$$E_\phi = j\frac{k_0 h w E_0 e^{-jk_0 r}}{\pi r}\sin\theta\cos\left(\frac{k_0 \ell_{eff}}{2}\sin\theta\sin\phi\right)\frac{\sin\left(\frac{k_0 h}{2}\sin\theta\cos\phi\right)}{\frac{k_0 h}{2}\sin\theta\cos\phi}\frac{\sin\left(\frac{k_0 w}{2}\cos\theta\right)}{\frac{k_0 w}{2}\cos\theta}. \tag{1.17}$$

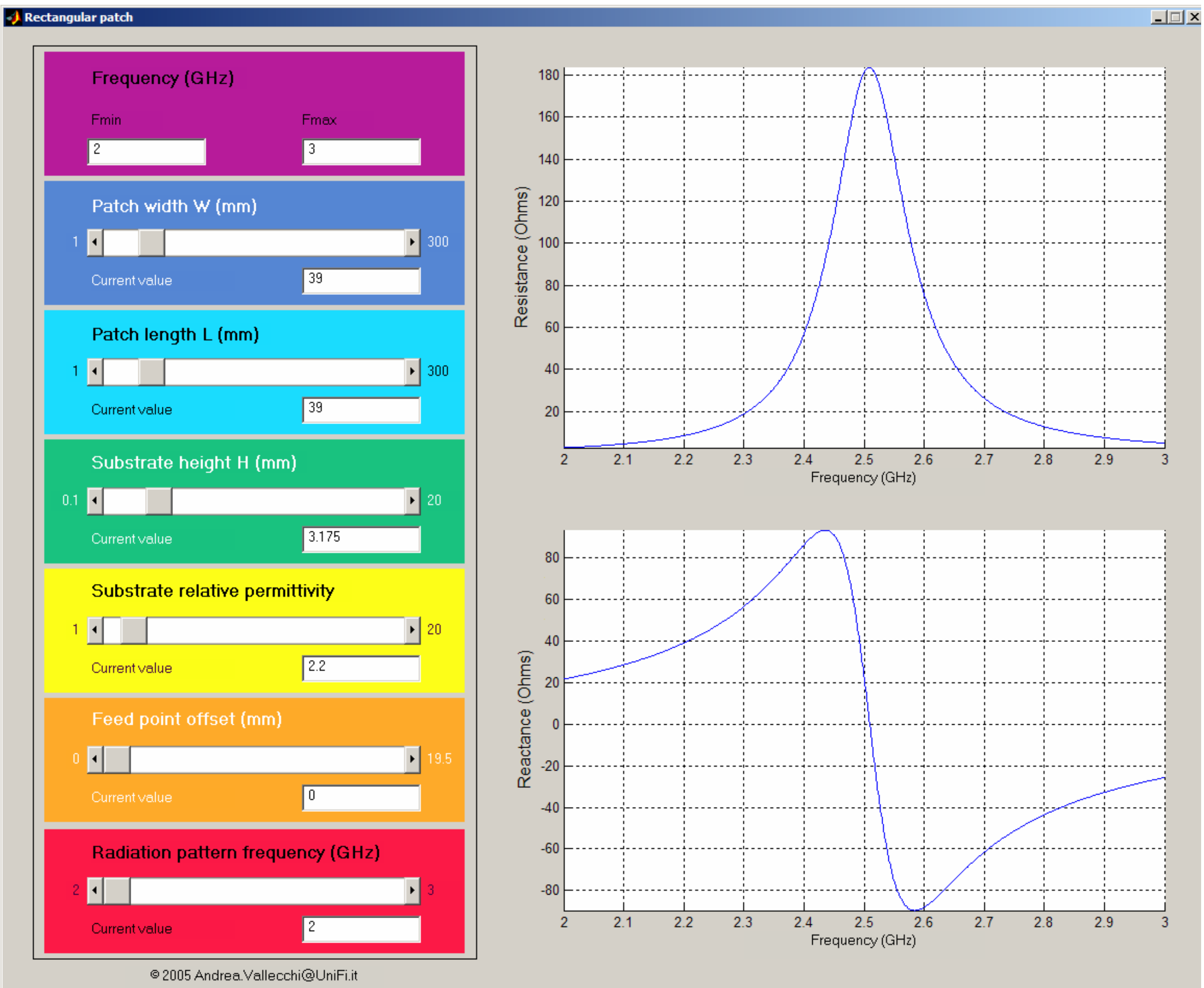

Figure 3. Screen-shot of the MATLAB application for the analysis of rectangular patches.

MATLAB script for the analysis of rectangular patches

On the basis of the theory outlined in the previous section, a MATLAB application has been developed to calculate the input impedance and radiation pattern of rectangular microstrip patches. The front panel of the application is shown in Figure 3. The geometrical and electrical quantities characterizing the patch can be directly type into the different editable fields of the application interface or continuously adjust by means of slider type controls. In particular, this latter possibility allows one to analyze how the various parameters involved in the design affect the performance of this radiating element and to highlight the trends associated with their change.

As an example, a microstrip antenna element is designed to resonate at a frequency f_0 = 2.4 GHz. The antenna is realized on a Arlon DiClad 870 dielectric substrate ($\varepsilon_r = 2.33$) of thickness h = 1.57 mm. At the resonant frequency, it is assumed that the desired input impedance is 120 Ohms.

The design consists of the following steps:

1. From relations (1.1) and (1.4) the width of the element is chosen as: $w = 60\lambda_0 / R_{in}$ = 62.5 mm.
2. To determine the resonant length of the antenna element, the effective permittivity has to be computed. This is done through (1.6) which gives ε_{reff} = 2.26. Then, the resonant length of the antenna element can be approximated as: ℓ = 0.48 $\lambda_0 \sqrt{\varepsilon_{reff}}$ = 40 mm.
3. Next, the characteristic impedance Z_c of a microstrip transmission line of width w is evaluated according to (1.7): it is obtained Z_c = 5.76 Ohms.

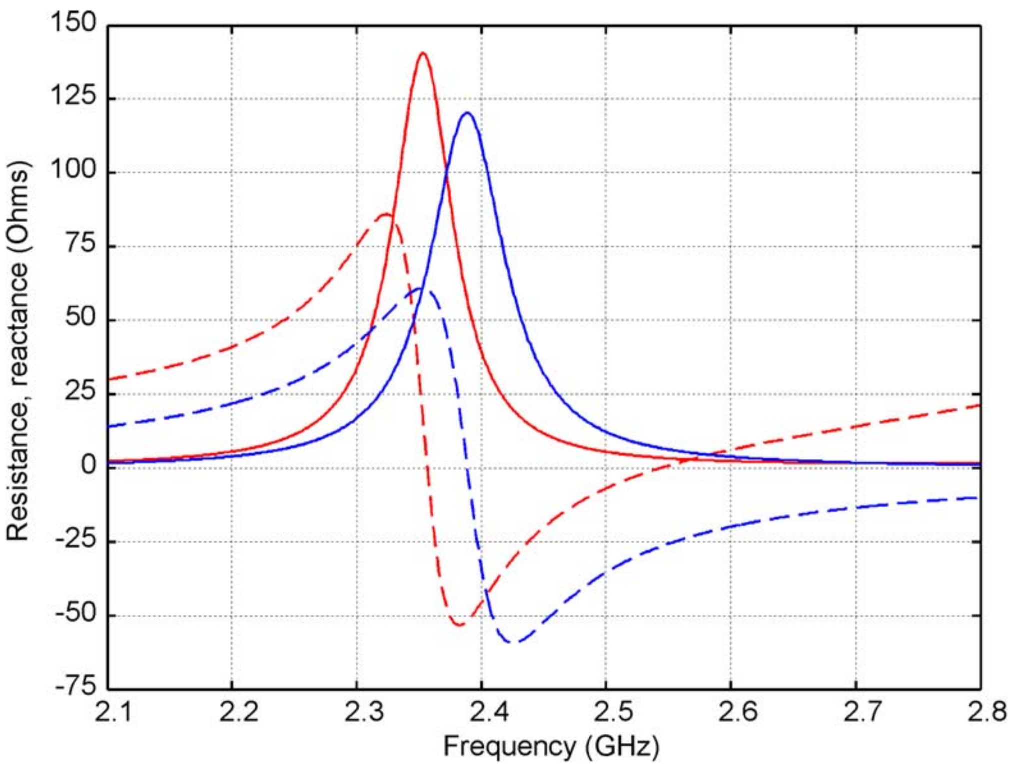

Figure 4. Input resistance (solid lines) and reactance (dashes lines) of a rectangular patch (ℓ=40 mm, w= 62.5, h=1.57, $\varepsilon_r = 2.33$) as computed by the transmission line model (blue lines) and the MoM (red lines).

The real and imaginary parts of the input impedance can be calculated through the developed MATLAB program and are shown in Figure 4 as a continuous and dashed blue line, respectively. The results obtained by using the commercial software ENSEMBLE, which is based on the mixed potential integral equation formulation in conjunction with the Method of Moments (MoM), are used as a reference and plotted in red in Figure 4 for reason of comparison.

It is observed that the resonant frequency as predicted by the transmission line model is a little shifted upwards with respect to the MoM data; furthermore, the magnitude of the resonant resistance is about 25% lower than the MoM result. However, it appears that the approximation provided by the transmission line model is sufficiently accurate to help in the first dimensioning of the radiating element.

The input impedance can be also evaluated at an arbitrary section of the patch by setting an appropriate value for the feed point distance from the patch edge in the corresponding editable field or slider control.

By varying the height of the antenna substrate or the permittivity of the dielectric material, the effects on the impedance plot and the bandwidth can be observed.

Finally, the MATLAB application allows the visualization of the patch radiation pattern by selecting the relevant frequency at the bottom of the programme window. In this respect, a good agreement with the results derived by a rigorous numerical approach is obtained, as testified by the principal plane pattern polar plots reported in Figure 5.

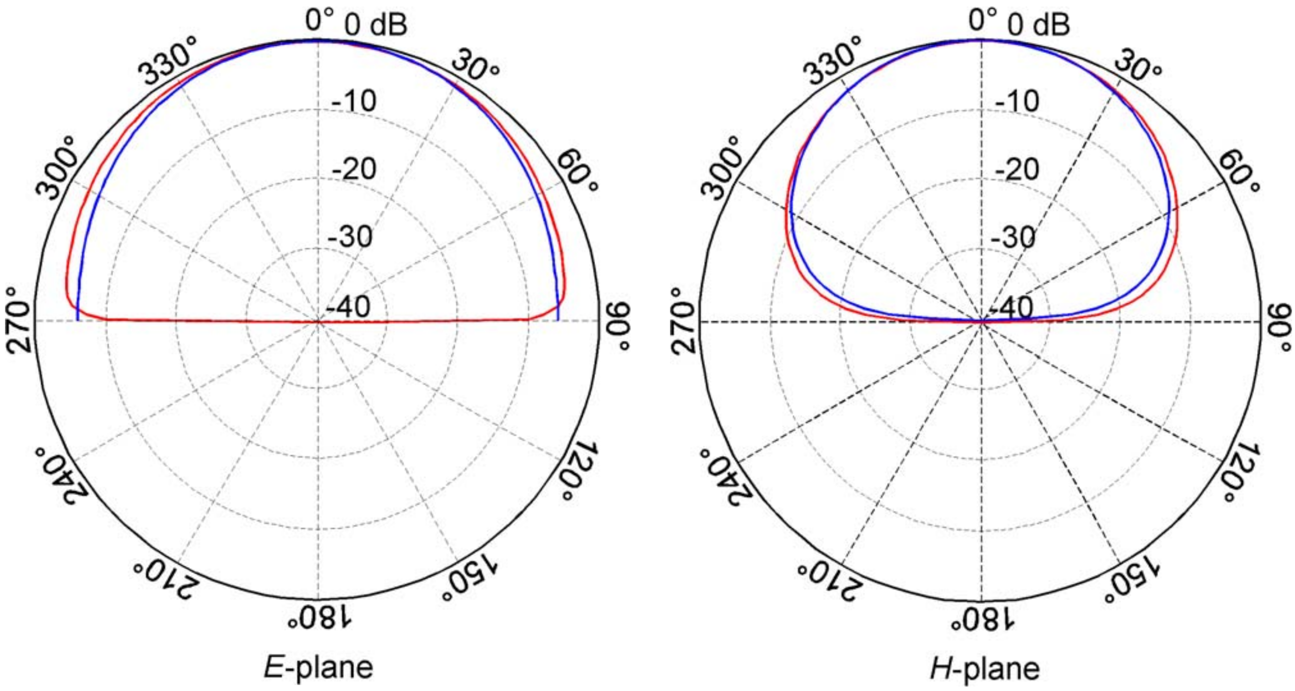

Figure 5. Principal plane radiation patterns of a rectangular patch (ℓ=40 mm, w= 62.5, h=1.57, $\varepsilon_r = 2.33$) as computed by the transmission line model (blue lines) and the MoM (red lines) at f = 2.38 GHz.

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